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Implementation of NBCC Torsional Provisions for Buildings without Locating Centres of Stiffness

R.K. Goel[1]

ABSTRACT

Static lateral force procedure of National Building Code of Canada (NBCC) requires that a torsional moment, in addition to the story shear, be included in the analysis of asymmetric-plan buildings. The torsional moment is specified as story shear times the design eccentricity; the latter depends on the static eccentricity. To implement this procedure of NBCC, it seemed necessary in the past to determine the static eccentricity, an often confusing and cumbersome procedure. This paper presents a new approach for implementing the NBCC procedure without the need for calculating the static eccentricity. The presented procedure combines the results from three static analyses that can be implemented directly on most commercially available computer programs for analysis of multi-story buildings. The new procedure is illustrated with a numerical example.

INTRODUCTION

Static force procedure of NBCC-1990 requires that a story torsional moment equal to the story shear times the design eccentricity be considered along with the story shear for analysis of asymmetric-plan buildings. The design eccentricity, e_{xj}, at the jth level is specified as

$$e_{xj} = 1.5\ e_j + 0.1\ D_{nj} \tag{1a}$$

$$e_{xj} = 0.5\ e_j - 0.1\ D_{nj} \tag{1b}$$

where e_j = static eccentricity at the jth level and D_{nj} = floor plan dimension of the building perpendicular to the direction of ground motion. For each structural element, the design eccentricity value leading to the larger design force is to be used.

The loads used in analysis of buildings by standard computer programs are the floor forces and floor torques, not the story shears and the story torsional moments. In such analysis, the floor torque is often computed as a product of the floor force and the design eccentricity of Eq. (1) with e_j defined as distance between the floor center of mass (CM) and the floor center of stiffness (CS) or center of rigidity (CR).

To implement the NBCC provision, it seemed necessary in the past to determine the locations of the CSs, an often cumbersome and confusing process. Therefore, a new approach was developed that avoids explicit determination of the CSs yet leads to results identical to those obtained by the procedure in which CSs were computed explicitly (Goel and Chopra 1993). This paper presents implementation of the NBCC-1990 torsional provisions using the new approach. Various steps of this approach are summarized first followed by a conceptual explanation of the approach using principle of superposition. Finally, a

[1] Assistant Professor, Department of Civil and Environmental Engineering, Syracuse University, Syracuse, NY 13244-1190.

numerical example is presented to demonstrate how to implement the NBCC torsional provision for asymmetric-plan systems.

ANALYSIS WITHOUT USING CENTERS OF STIFFNESS

The new approach to implement the NBCC static-force procedure for asymmetric-plan buildings combines, according to a simple rule, the results of three sets of analyses. In each of these analyses, the forces are applied at the floor CMs. The three analyses are summarized in steps 1-3, their superposition in step 4, and the selection of the design value in step 5.

1. With the code-specified lateral forces F_{yj} applied at the floor CMs, analyze the building restricted to deform only in the direction of applied load (Figure 1). This analysis can be implemented in standard computer program for building analysis by constraining the floor rotations. The resulting value of the desired response (force or deformation) is $r^{(1)}$.

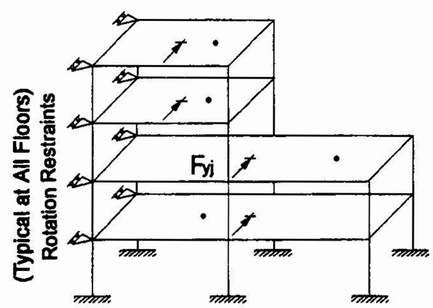

Figure 1. Step 1 in analysis by the new approach

2. With the code-specified lateral forces F_{yj} applied at the floor CMs, analyze the asymmetric-plan building as a three-dimensional system to obtain the value $r^{(2)}$ of the desired response (Figure 2).

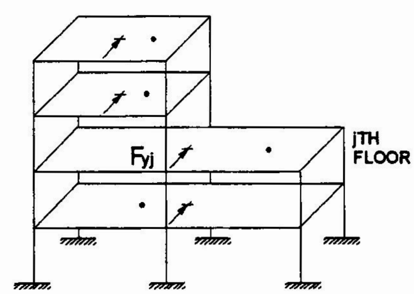

Figure 2. Step 2 in analysis by the new approach

3. Analyze the asymmetric-plan system for the code-specified floor torques $\beta b_j F_{yj} = 0.1 D_{nj} F_{yj}$ to obtain the value $r^{(3)}$ of the desired response (Figure 3).

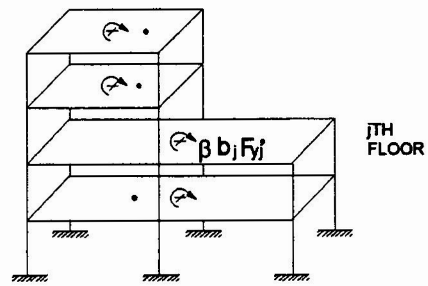

Figure 3. Step 3 in analysis by the new approach

4. Obtain the responses $r^{(a)}$ and $r^{(b)}$ associated with design eccentricities of Eqs. (1a) and (1b), respectively, by combining $r^{(1)}$, $r^{(2)}$, and $r^{(3)}$ as follows:

$$r^{(a)} = (1-\alpha) r^{(1)} + \alpha r^{(2)} + r^{(3)} \tag{2}$$

$$r^{(b)} = (1-\delta) r^{(1)} + \delta r^{(2)} - r^{(3)} \tag{3}$$

Since $\alpha = 1.5$ and $\delta = 0.5$ in NBCC, Eqs. (2) and (3) become

$$r^{(a)} = -0.5 r^{(1)} + 1.5 r^{(2)} + r^{(3)} \tag{4}$$

$$r^{(b)} = 0.5 r^{(1)} + 0.5 r^{(2)} - r^{(3)} \tag{5}$$

In each of Eqs. (2) to (5), the sign of $r^{(3)}$ should be selected as follows. If $r^{(1)}$ is less than $r^{(2)}$ in magnitude, the sign of $r^{(3)}$ should be such that it increases the magnitude of the sum of the first two terms in Eqs. (2) and (4); conversely it reduces the magnitude obtained from the sum of first two terms in Eqs. (3) and (5). On the other hand, if $r^{(1)}$ is greater than $r^{(2)}$ in magnitude, the sign of $r^{(3)}$ should be taken such that it increases the magnitude obtained by the sum of the first two terms in Eqs. (3) and (5) and reduces the magnitude in Eqs. (2) and (4).

5. The design value of the desired response is the larger of the two values $r^{(a)}$ and $r^{(b)}$.

CONCEPTUAL EXPLANATION OF THE NEW APPROACH

As mentioned previously, implementation of NBCC torsional provisions is equivalent of applying the lateral floor force F_{yj} at a distance equal to the design eccentricity e_{xj} from the CS at the jth level, in which e_{xj} is given by either Eq. (1a) or (1b). For simplicity, let us re-write Eqs. (1a) and (1b) as

$$e_{xj} = \gamma e_j + \beta b_j \tag{6}$$

in which $\gamma = 1.5$ and $\beta b_j = 0.1 D_{nj}$ to arrive at Eq. (1a), and $\gamma = 0.5$ and $\beta b_j = -0.1 D_{nj}$ to arrive at Eq. (1b).

For buildings with rigid diaphragms, this load condition is equivalent to superposition of three load cases (Figure 4): (1) Lateral forces F_{yj} ($j = 1, 2, ..., N$) at the floor CSs; (2) floor torques = $\gamma e_j F_{yj}$; and (3) floor torques = $\beta b_j F_{yj}$. The first load case of Figure 4 is equivalent to step 1 of the new approach (Figure 1) because the lateral stiffness and the lateral forces are the same in the two cases and the floor rotations are absent in both cases: they have been prevented in step 1 of the new approach and do not occur in the first load case of Figure 4 because forces are applied at the CSs. Step 3 of the new approach (Figure 3) is equivalent to the third load case of Figure 4 because in both cases the forces applied are the same pure floor torques.

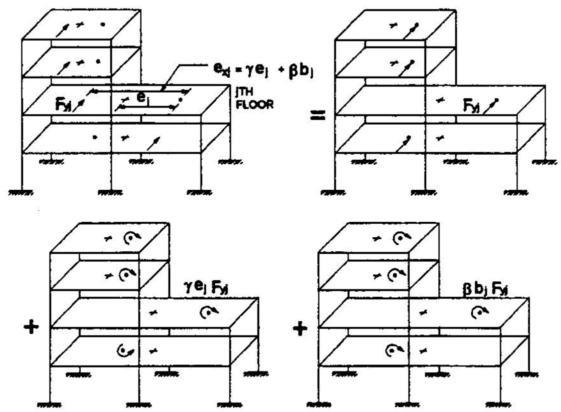

Figure 4. Conceptual explanation of the new approach

The second load case of Figure 4 includes floor torques = $\gamma e_j F_{yj}$ which can be expressed as superposition of three load cases (Figure 5): (1) Lateral forces $= \gamma F_{yj}$ at the CSs; (2) floor torques = $\gamma e_j F_{yj}$; and (3) lateral forces $= -\gamma F_{yj}$ at the CSs. The first two load cases combined are equivalent to the application of the lateral forces $= \gamma F_{yj}$ at the CMs. Consequently, the second load case of Figure 4 is equivalent to γ times the results of step 2 minus γ times the results of step 1 of the new approach.

Restating the conclusions of the preceding paragraphs, the response due to application of the lateral forces at a distance equal to the design eccentricity from the CSs is obtained as

$$r = r^{(1)} + \gamma (r^{(2)} - r^{(1)}) \pm r^{(3)} \tag{7}$$

or

$$r = (1-\gamma)\, r^{(1)} + \gamma\, r^{(2)} \pm r^{(3)} \tag{8}$$

which is a combined version of Eqs 2 and 3.

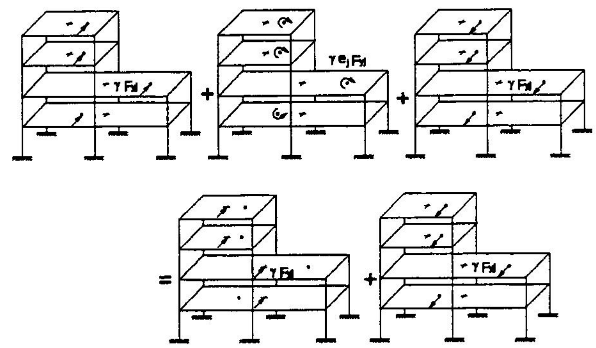

Figure 5. Second load case of Figure 4 as a combination of steps 1 and 2 of the new approach

NUMERICAL EXAMPLE

To illustrate the procedure presented in this paper, consider a four-story building having three frames A, B, and C spanning in the y-direction connected through rigid floor diaphragms (Figure 6); the building is symmetric in the x-direction. All beams in the frames are identical, with moment of inertia $I_b = 0.3\ m^4$. The column moment of inertia is $0.1\ m^4$ for frame B and $0.05\ m^4$ for frames A and C. The columns are assumed to be axially rigid. The floor weights are 20 kN for each of the bottom two floors and 10 kN for the top two floors. This example building is the same as that used by Tso (1990).

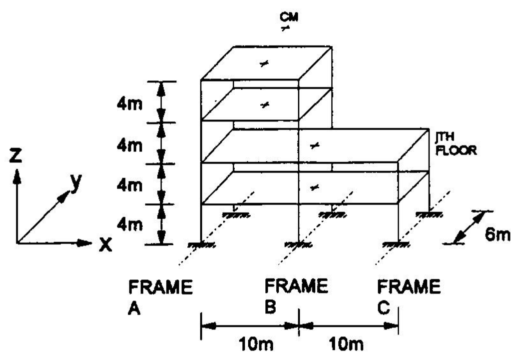

Figure 6. Example building (After Tso, 1990)

This example building has large discontinuity in stiffness and mass between the second and third stories. Consequently, it may not satisfy the code criteria for the equivalent static procedure to be applicable. This building is chosen here, however, for the purpose of illustrating the new procedure and should not be considered as an example where the NBCC static procedure is necessarily applicable.

The example building is designed as per the seismic provisions of 1990 NBCC. The fundamental period of the building is calculated as $0.1N = 0.1 \times 4 = 0.4$ sec. For $Z_a / Z_v > 1$ and $T = 0.4$ sec, the seismic response factor S =4.2-8.4(T-0.25) = 2.94. The seismic importance factor I and foundation factor F are selected as 1 and the building is assumed to be located at a site with zonal velocity ratio of 0.4. For the total building weight of 60 kN, the equivalent lateral elastic seismic force V_e = 70.56 kN. This elastic level force is reduced by considering the force modification factor R = 4 for ductile reinforced concrete moment resisting frames and the level of protection factor U = 0.6 to obtain the minimum lateral design seismic force of V = 10.58 kN. The variation of lateral forces over the height of the building is computed as illustrated in Table 1.

Table 1. Height wise distribution of lateral forces

Floor j (1)	Height h_j (2)	Weight w_j (3)	$w_j h_j$ (4)	$F_{yj} = \frac{w_j h_j}{\Sigma w_j h_j} V$ (5)
4	16	10	160	3.26
3	12	10	120	2.44
2	8	20	160	3.26
1	4	20	80	1.63

By using the procedure outlined by Goel and Chopra (1993), the static eccentricities for this building when subjected to the NBCC specified height wise distribution of lateral forces (column 5 of Table 1) are computed as $e_4 = 1.00$ m, $e_3 = 1.16$ m, $e_2 = 7.59$ m, and $e_1 = -1.34$ m. Using the traditional analysis approach in which the eccentricities are explicitly computed gives the design forces summarized in Table 2. The shear values for analysis 1 correspond to Eq. (1a) and those for analysis 2 correspond to Eq. (1b). These values are included to demonstrate that the new procedure would indeed lead to design values identical to those obtained by the traditional approach.

Table 2. Shears in columns of the example building by the traditional approach

Frame (1)	Floor j (2)	Shear: Analysis 1 (kN) (3)	Shear: Analysis 2 (kN) (4)	Design Shear (kN) (5)
A	4	0.90	0.60	0.90
	3	1.57	1.04	1.57
	2	2.39	1.33	2.39
	1	2.61	1.38	2.61
B	4	0.73	1.03	1.03
	3	1.28	1.81	1.81
	2	1.92	1.85	1.92
	1	2.47	2.48	2.48
C	2	0.17	1.31	1.31
	1	0.22	1.44	1.44

The results for the same building obtained by the new procedure are summarized in Table 3. The shear forces in the columns of the building from the three analyses, summarized in steps 1, 2, and 3, are presented in columns 3, 4, and 5, respectively of Table 3. Combination of these three values in accordance with Eqs. (4) and (5) provide $V_j^{(a)}$ and $V_j^{(b)}$ in columns 6 and 7, respectively. The larger of these two values gives the design shear in column 8 of Table 3.

Table 3. Shears in columns of the example building by the new approach

Frame (1)	Floor j (2)	$V_j^{(1)}$ (kN) (3)	$V_j^{(2)}$ (kN) (4)	$V_j^{(3)}$ (kN) (5)	$V_j^{(a)}$ (kN) (6)	$V_j^{(b)}$ (kN) (7)	Shear (kN) (8)
A	4	0.65	0.75	0.10	0.90	0.60	0.90
	3	1.12	1.30	0.18	1.57	1.03	1.57
	2	1.25	1.86	0.23	2.39	1.33	2.39
	1	1.41	1.99	0.32	2.60	1.38	2.60
B	4	0.98	0.88	-0.10	0.73	1.03	1.03
	3	1.73	1.55	-0.18	1.28	1.82	1.82
	2	1.86	1.88	-0.03	1.91	1.85	1.91
	1	2.47	2.47	0.00	2.47	2.47	2.47
C	2	1.37	0.74	-0.25	0.17	1.31	1.31
	1	1.41	0.83	-0.32	0.22	1.44	1.44

To illustrate implementation of the combination rule in the new procedure, let us calculate values of forces $V_4^{(a)}$ and $V_4^{(b)}$ in frames A and B. For frame A, $V_4^{(1)}$ = 0.65 kN is less than $V_4^{(2)}$ = 0.75 kN. Thus, the algebraic sign of $V_4^{(3)}$ = 0.10 kN is selected such that it increase the magnitude of the sum of the first two terms in Eq. (4) and reduces the magnitude of the sum of the first two terms in Eq. (5). This leads to values of:

$V_4^{(a)} = -0.5V_4^{(1)} + 1.5V_4^{(2)} + V_4^{(3)} = -0.5 \times 0.65 + 1.5 \times 0.75 + 0.10 = 0.90\text{kN}$ and

$V_4^{(b)} = 0.5V_4^{(1)} + 0.5V_4^{(2)} - V_4^{(3)} = 0.5 \times 0.65 + 0.5 \times 0.75 - 0.10 = 0.60\text{kN}$

in columns 6 and 7, respectively, of Table (3). The larger of these two values = 0.90 kN is the design value presented in column 8 of this Table.

For frame B, $V_4^{(1)}$ = 0.98 kN is larger than $V_4^{(2)}$ = 0.88 kN. Thus the algebraic sign of $V_4^{(3)}$ = 0.10 kN is selected such that it reduces the magnitude of the sum of the first two terms in Eq. (4) and increases the magnitude of the sum of the first two terms in Eq. 5. This leads to values of:

$V_4^{(a)} = -0.5V_4^{(1)} + 1.5V_4^{(2)} - V_4^{(3)} = -0.5 \times 0.98 + 1.5 \times 0.88 - 0.10 = 0.73\text{kN}$ and

$V_4^{(b)} = 0.5V_4^{(1)} + 0.5V_4^{(2)} + V_4^{(3)} = 0.5 \times 0.98 + 0.5 \times 0.88 + 0.10 = 1.03\text{kN}$

in columns 6 and 7, respectively, of Table (3). The larger of these two values = 1.03 kN is the design value presented in column 8 of this Table.

For the example building selected in this paper, the shears in columns computed from the new approach are the same as those obtained by the procedure using the explicit locations of CSs as demonstrated by the identical results in columns 3-5 of Table 2 and columns 6-8 of Table 3.

CONCLUSIONS

A new approach is presented for implementing static lateral force procedure of 1990 NBCC for asymmetric-plan buildings without locating the CSs, an often confusing and cumbersome procedure. The presented procedure combines the results from three static analyses that can be implemented directly on most commercially available computer programs for analysis of multi-story buildings. The work presented in this paper should dispel the long-held view that locations of the CSs must be determined to implement the code procedure, thereby removing one of the major difficulties in building analysis. A numerical example is presented to illustrate implementation of the NBCC procedure using the new approach.

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